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One of the significant challenges to successful formation flight of spacecraft is maintenance of the formation, i.e., control of the motion of the individual spacecraft to maintain the overall formation. This includes both stabilization of a given formation and reconfiguring of the formation- While the dynamics and control a single spacecraft; is well understood, a formation of spacecraft effectively acts a deformable body due to control forces which restore it to its desired formation, As a deformable body, the formation is capable of exhibit complex dynamic behavior. Effective control strategies must exploit this behavior as well as the natural dynamics of the system to achieve goals such as formation error minimization and minimal fuel consumption during formation reconfiguration. An additional concern is the impact a decentralized control structure would have control algorithm design and formation controllability. Spacecraft dynamics are mechanical, meaning they admit a Lagrangian or Harniltonian formulation. We have investigated the dynamics and control of formation flight by exploiting the mechanical structure of the dynamical systems in conjunction with proven methods of linearization and structured uncertainty Our work built on previous analytical tools developed at Caltech, such as the energy-momentum method for assessing the stability of a mechanical system, as well as methodologies for control of mechanical systems. One important property of mechanical systems is the ability of small changes in the internal shape of the system to effect global motion of the system.

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# FORMATION FLIGHT OF MICRO-SATELLITE CLUSTERS

AFOSR GRANT F49620-99-1-0190

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**Final Report**

## 1 Background and Objectives

One of the significant challenges to successful formation flight of spacecraft is maintenance of the formation, i.e., control of the motion of the individual spacecraft to maintain the overall formation. This includes both stabilization of a given formation and reconfiguring of the formation. While the dynamics and control a single spacecraft is well understood, a formation of spacecraft effectively acts a deformable body due to control forces which restore it to its desired formation. As a deformable body, the formation is capable of exhibit complex dynamic behavior. Effective control strategies must exploit this behavior as well as the natural dynamics of the system to achieve goals such as formation error minimization and minimal fuel consumption during formation reconfiguration. An additional concern is the impact a decentralized control structure would have control algorithm design and formation controllability.

Spacecraft dynamics are mechanical, meaning they admit a Lagrangian or Hamiltonian formulation. We have investigated the dynamics and control of formation flight by exploiting the mechanical structure of the dynamical system in conjunction with proven methods of linearization and structured uncertainty. Our work built on previous analytical tools developed at Caltech, such as the energy-momentum method for assessing the stability of a mechanical system, as well as methodologies for control of mechanical systems. One important property of mechanical systems is the ability of small changes in the internal shape of the system to effect global motion of the system. Exploiting this phenomenon, termed *geometric phase*, in conjunction with the inherent nonlinear instability of a formation in certain regimes of its cluster of orbits, has yielded methods for reconfiguring the formation that make use of very small amounts of energy. Another relevant area of research at Caltech is in trajectory generation for mechanical systems, which we have further developed and applied in this project.

The specific objectives of this work were to (1) explore the dynamics of multiple spacecraft flying in nearby orbits, (2) evaluate the general power requirements and limitations to maintaining or reconfiguring a rigid formation, (3) produce a mathematical model of formation motion which captures the relevant effects, (4) propose formation stabilization strategies and (5) design formation reconfiguration algorithms. The specific approaches that were pursued were aimed at exploiting the natural dynamics and the mechanical structure of the system in conjunction with proven methods in nonlinear dynamics, robust control, and geometric control.

# 2 Accomplishments

## 2.1 Nonlinear Dynamics for Formation Flight.

**Poincaré Map and Pseudo-circular Orbits for Formation Flight.** We used the basic tools of dynamical systems theory to tackle the problem of formation flight including the $J_2$ (bulge of the Earth) perturbation effects. Since a Poincaré maps can be used to investigate the global orbit structure of a nonlinear system, we consider it to be an essential tool for studying the formation flight of micro-satellites. We have been able to use it to identify candidate reference orbits for which the nearby dynamics may support formation flight.

- Following Broucke [1994], we have used the $z$-axis symmetry of the $J_2$ problem (where the longitude variable $\phi$ is ignorable and the $z$ component of the angular momentum is conserved) to reduce the equations of motion into two second order equations in the $(\rho, z)$ variables. Here, $(\rho, z)$ are the rectangular coordinates of the satellite in the co-rotating meridian plane.
- Since energy is conserved, the constant energy surface is three dimensional and the hyperplane $z = 0$ can be used as the transversal plane to obtain the two dimensional Poincaré section. As the $z$ component of angular momentum (which is related to the inclination) is varied from 1 to 0 (from equator to polar), a number of interesting bifurcations take place, especially around the critical inclination.
- By studying these Poincare sections and looking for stable fixed points, we can find the pseudo-circular orbits (which correspond to the fixed points on the Poincaré map) whose nearby (quasi-periodic) orbits can be used for formation flight. Preliminary numerical exploration using Matlab leads us to believe that a cluster of satellites on these quasi-periodic orbits will stay close to each other for a long time and the pseudo-circular orbits can be used as candidate reference orbits for formation flight.

**Satellite Dynamics and the Kepler-$J_2$ Problem.** We have developed a geometric mechanical framework for the study of Kepler-$J_2$ Problem, using a variety of tools, such as averaging, symmetry reduction, geometric phases and bifurcation. These results will shed light on the Poincaré map method used above in finding candidate reference orbits. In doing this, it is important to remember that the $J_2$ perturbation terms cause a secular drifts in the longitude of the ascending node $\Omega$ and the argument of perigee $\omega$. But other orbital elements do not change on average. This suggests the use of averaging method.

- By using Moser's averaging method [1970] (with respect to the relatively faster Kepler motion), the $J_2$ perturbed Hamiltonian becomes $S^1$-invariant (due to time-symmetry). After double reductions (a $S^1$ time-symmetry and an axial $S^1$ symmetry around $z$-axis), we obtained a reduced Hamiltonian flow parameterized by the energy and the $z$-component of the angular momentum.
- This flow has interesting bifurcations which are qualitatively similar to the results in Coffrey *et al.* [1994]. In fact, these bifurcation diagrams look very much like those obtained in Broucke [1994] using the Poincaré map method.
- We can derive the well-know formulas for the drift rate of the mean orbital elements $\bar{\omega}$ and $\bar{\Omega}$. More interestingly, we are able to interpret the drift in $\bar{\Omega}$ as a geometric phase caused by the cyclic motion of $\bar{\omega}$ in the reduced space and obtain a formula for the increment of $\bar{\Omega}$ in one revolution of $\bar{\omega}$.

We still need to link the reduction picture with the Poincaré map picture of Broucke [1994] and get a deeper understanding of how the fixed points of the Poincaré map for the meridian dynamics is related to the equilibria of our reduced Hamiltonian and how to use the pseudo-circular orbits for formation flight.

## 2.2 Control and Optimal Control

Using dynamical systems theory, we have found candidate reference orbits whose nearby orbits may support formation flight. This result allows the use of linear control techniques for maintenance. We have also successfully applied optimal control techniques to larger dynamic motions, for sample problems.

**Merging Optimal Control with Dynamical Systems Theory.** In formation reconfiguration, larger dynamic motions are involved: imagine asking the formation to point to a different spatial direction. Fuel useage is also potentially critical. Having approximately optimal solutions is one way to approach the problem. Hence, techniques of optimal control theory and its related numerical method are important and useful.

As usual, for any numerical algorithm, a good first guess is vital, especially if the problem is very sensitive numerically. Dynamical systems theory can provide geometrical insight into the structure of the problem and even good approximate solutions. For example, in finding low-thrust optimal transfers to halo orbits in the Sun-Earth system, it is important to know that the invariant manifolds of the halo orbits extend to the vicinity of the Earth and any trajectory on these manifolds can be used as a super-highway for free rides to and from the halo orbits. Clearly, this theoretical insight and its derivative numerical tools can aid in the construction of superior first guesses that lead to a convergent solution.

**Optimal Control and Trajectory Correction Maneuvers.** The above ideas have been put to an initial test in a closely related joint work effort with JPL and UCSB (the Computational Science and Engineering Group). See Serban *et al.* [2000]. This paper addresses the computation of the required trajectory correction maneuvers (TCM) for the Genesis mission[1] to compensate for the launch velocity errors introduced by inaccuracies of the launch vehicle.

Right after launch, before the spacecraft initial checkout activities have been completed, the performance of an early maneuver such as TCM1 is both difficult and risky. It is desirable to delay TCM1 as long as possible. In fact, Genesis would prefer TCM1 be performed at 2 to 7 days after launch, or even later. However, beyond launch + 24 hours, the correction $\Delta V$ based on traditional linear analysis can become prohibitively high. The desire to increase the time between launch and TCM1 drives one to use a nonlinear approach, based on combining dynamical systems theory with optimal control techniques. Two similar but slightly different approaches have been explored. (1) HOI technique: use optimal control techniques to re-target the halo orbit with the original nominal trajectory as the initial guess. (2) MOI technique: target the stable manifold. Both methods yield good results using the software COOPT which is based on the direct method and developed at UCSB.

We feel that COOPT or similar software and the methods of optimal control and dynamical systems can be used for many missions in the future. It will be an essential tool for designing formation flights near a halo orbit or for earthbound satellites.

[1] A sample and return mission to the first Lagrange point, to collect particles from the solar wind

**Formation Reconfiguration for Earthbound Satellites.** The overall approach we have taken in solving the micro-satellite problem is to break the control into two modes: maintenance and reconfiguration. By exploiting the natural dynamics near a good candidate reference orbit, we expect that the linear control techniques will be sufficient for formation maintenance. But for larger motions such as formation reconfiguration, optimal control and other nonlinear techniques will most likely be needed.

We have solved the following simple triangular formation problem using the Nonlinear Trajectory Generation (NTG) software developed at Caltech (under the AFOSR PRET program) which is based on direct method optimization and which make use of the mechanical structure of of the problem:

- Find a trajectory which minimizes the control for three micro-satellites such that at the final configuration (and at a fixed time), the micro-satellites (or more accurately, their projections on the plane normal to the line of sight) form an equilateral triangle and will remain on a circle, indefinitely, without control force.

The fact that each micro-satellite is fully-actuated makes this optimal control problem particularly easy for NTG since there are no resulting system dynamics to adjoin as in the usual optimal control problem using the Maximum Principle. Verification and investigation of the constellation trajectories is made by an $N$-satellite simulator/animator developed specifically for the micro-satellite project.

The sample problem used only Hill's equations (linearized about a circular orbit and without $J_2$ effect) for the system dynamics. While this is not realistic, it is a demonstration that the software is effective. Besides, since the reconfiguration usually takes no more than a few revolutions around the earth, the $J_2$ effect should be small and this trajectory can be used as a first guess in any subsequent more realistic optimization. In the immediate future, we would like to solve similar reconfiguration problem using either the linearized or even the full non-linear equations about a nominal trajectory such as the candidate reference orbits mentioned about.

**Other Related Work on Optimal Control.** Another area that we investigated was the computation of fuel-optimal formation reconfiguration control laws. Our approach wass to better understand the relationship between standard optimal control formulations and the mechanical structure of satellite dynamics. The general theory of optimal control centers on first-order systems, while satellite dynamics possess a much richer structure which can be exploited in understanding the nature of optimal trajectories and in computing them more efficiently. We have explored the structure of the optimal controls for mechanical systems and the determined some linkages between the geometry of the Riemannian metric corresponding to kinetic energy and the geometry of the equations governing the optimal control. In addition, we explored the use of time-scaling to reduce the dimension of certain classes of optimal control problems for mechanical systems and to produce feedback controllers that generate the optimal solutions. Future work will include advancing the theory further and developing optimal trajectory calculation tools based on the theory.

**Cooperative Control.** Micro-satellite formations have the property that the individual satellites, while decoupled dynamically, are coupled through the task they are asked to perform. In general, each satellite will not be able to directly sense the state of the entire formation, nor may they be able to communicate directly with the entire formation. Thus, each satellite must locally implement a control law based on limited sensed information through which the formation as a whole will achieve its objective.

We have recently begun investigating relevant issues in cooperative control. Our viewpoint is that satellite sensing and communication represent flow of information throughout the formation, and that the topology of those information flows (which may not be identical) has important ramifications for controller design. In this project, we considered primarily "low-level" controls issues, seeking to answer questions such as: how does the topology of the sensed information flow impact local controller stability? Is it possible to design controllers which will stabilize the formation regardless of the sensing topology? Of the many controller design methodologies, which are relevant to this problem? What information should individual satellites broadcast to improve overall performance? What are the tradeoffs between transmission bandwidth and formation performance? To what extent is global information about the formation needed by individual satellites to execute tasks such as formation reconfiguration? When this is concluded, we plan to look into 'higher-level' control issues, such as: how do the individual satellites form a consensus about its own status within a decentralized setting such as this? How can the formation assign roles to individual satellites? These topics are relevant not only to satellite formation control, but to cooperative control of multi-vehicle systems generally.

**Lyapunov-Based Global Orbit Transfer.** We have studied the global orbit transfer between elliptic orbits about a spherical Earth, in which the final time is not specified and the injection point is free. We define the orbit at all times through the natural quantities of the angular momentum vector and the eccentricity vector. It is shown that every non-collision Keplerian orbit can be uniquely described by these two vectors.

We use the difference between current and desired final values of these vectors to define a Lyapunov function, and by driving this function to zero we effect the transfer. It can be shown that for any chosen target orbit, there is a neighborhood in which the control so defined will cause the orbit to change to the target, while avoiding collision orbits. It can also be shown that a path can be defined from any starting to any final elliptic orbit, using a finite number of intermediate orbits, such that the control law will accomplish the desired transfer.

The Lyapunov function might also be expanded to include the orbital energy explicitly, which would give a more general form, perhaps leading to a broader class of controllers. Moreover, there is also a need to extend this method to formation reconfiguration of a cluster of satellites.

## 3 Personnel Supported

| | |
|---|---|
| Faculty | Jerrold Marsden, Caltech<br>Richard Murray, Caltech |
| Staff researchers | David Chichka, Caltech |
| Postdoctoral fellows | Wang Sang Koon, Caltech<br>Nicolas Petit, Caltech/Ecole des Mines |
| Graduate students | Dong Eui Chang, Caltech<br>Alex Fax, Caltech<br>Amber Thweatt, Caltech<br>Mark Milam, Caltech |

## 4 Publications

- J. A. Fax [2001], Optimal and Cooperative Control of Vehicle Formations, PhD Dissertation, Control and Dynamical Systems, Caltech, 2001.
- W. S. Koon, J. E. Marsden, J. Masdemont and R. M. Murray [2001], J2 Dynamics and Formation Flight, AIAA Guidance, Navigation and Control Conference, 2001.
- M. B. Milam, N. Petit and R. M. Murray [2001], Constrained Trajectory Generation for Microsatellite Formation Flying, AIAA Guidance, Navigation and Control Conference, 2001.
- M. W. Lo, W. S. Koon, J. E. Marsden and R. M. Murray [2000], Formation Flight Near Libration Points, Survey and Recommendations, The Terrestial Planet Finder Study, April, 2000.
- R. Serban, W. S. Koon, M. W. Lo, J. E. Marsden, L. R. Petzold, S. D. Ross and R. S. Wilson [2000], Halo orbit mission correction maneuvers using optimal control, submited to *Automatica*.
- J. A. Fax and R. M. Murray [2000], Optimal Control of Affine Connection Control Systems: A Variational Approach, 2000 Conference on Decision and Control.
- J. A. Fax and R. M. Murray [2000], Finite-Horizon Optimal Control and Stabilization of Time-Scalable Systems. 2000 Conference on Decision and Control.

## 5 Interactions and Transitions

### Meetings and Conferences

In addition to presentation of results ant meetings and conferences, this project hosted a visit by Meir Pachter to Caltech (in April 2000) and participated in two TechSat 21 meetings. We also organized a workshop at the 2001 *SIAM Conference on Control and Its Applications*, which included 10 speakers working in the area of formation flight of microsatellite clusters.

### Transitions

The work in this program was used in a variety of followon and related activities. Two specific transitions are worthy of note:

- Orbit generation: this work in this grant was closely related to work supported by JPL on orbit generation for the Genesis Mission. The specific results preformed under this grant let to a deeper understanding of the dynamical systems analysis of spacecraft systems. Contact: Martin Low (JPL).
- Nonlinear trajectory generation (NTG): this grant supported the initial development of the NTG software package, which is now a central element in the DARPA Software Enabled Control (SEC) program and the DARPA Mixed Initiative Control of Automa-teams (MICA) program. Contacts: John Bay (SEC) and Sharon Heise (MICA).